Линейное прибл-е системы и одномерных УГД

$\frac{\partial \rho}{\partial t} + \rho \frac{\partial}{\partial x} v + v \frac{\partial}{\partial x} \rho = 0$; $\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} + \frac{1}{\rho} \frac{\partial p}{\partial x} = 0$; S - энтропия $\frac{\partial S}{\partial t} + v \frac{\partial S}{\partial x} = 0$

ур-е энергии в энтр-ой форме $\frac{dS}{dt} = 0$

$p(\rho, T)$, $S(p, T)$; $p_0, \rho_0, v_0 = 0$; покоящийся газ с пар-ми

вносим малые возм-я $\tilde{p}, \tilde{\rho}, \tilde{v}$; $\tilde{p} \ll p_0$, $\tilde{\rho} \ll \rho_0$;

$p = p_0 + \tilde{p}$, $\rho = \rho_0 + \tilde{\rho}$, $v = \tilde{v}$; $\frac{\partial \rho}{\partial t} + \rho \frac{\partial}{\partial x} v + v \frac{\partial}{\partial x} \rho = 0$; $\frac{\partial v}{\partial t} + v \frac{dv}{dx} + \frac{1}{\rho} \frac{\partial p}{\partial x} = 0$

$S = S_0$; $\frac{\partial S}{\partial x} = 0$; $S(p, T) \to T(S, p) \to T(S, \rho)$; $p(\rho, T) \to p(\rho, S) \to p(\rho, S_0)$

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial x} p(\rho, S_0) = \frac{\partial p}{\partial \rho} \cdot \frac{\partial \rho}{\partial x} + \frac{\partial p}{\partial S} \frac{\partial S}{\partial x} = \left.\frac{\partial p}{\partial \rho}\right|_{S = S_0} \frac{\partial \rho}{\partial x} = C_0^2 \frac{\partial \rho}{\partial x}; \quad \left.\frac{\partial p}{\partial \rho}\right|_{S = S_0} = C_0^2$$

$\rho = \rho_0 + \tilde{\rho}$, $p = p_0 + \tilde{p}$, $v = \tilde{v} + v_0 = \tilde{v}$ $\Rightarrow \frac{\partial}{\partial t}(\rho_0 + \tilde{\rho}) + (\rho_0 + \tilde{\rho}) \frac{\partial}{\partial x} \tilde{v} + \tilde{v} \frac{\partial}{\partial x}(\rho_0 + \tilde{\rho}) = 0$

$$\frac{\partial}{\partial t} \rho_0 + \frac{\partial}{\partial t} \tilde{\rho} + \rho_0 \frac{\partial}{\partial x} \tilde{v} + \cancel{\tilde{\rho} \frac{\partial}{\partial x} \tilde{v}} + \tilde{v} \frac{\partial}{\partial x} \rho_0 + \cancel{\tilde{v} \frac{\partial}{\partial x} \tilde{\rho}} = 0$$

$$\frac{\partial}{\partial t} \tilde{\rho} + \rho_0 \frac{\partial}{\partial x} \tilde{v} = 0; \quad \frac{\partial}{\partial t} \tilde{\rho} + \rho_0 \operatorname{div} \tilde{v} = 0$$

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} + \frac{1}{\tilde{\rho}} C_0^2 \frac{\partial \rho}{\partial x} = 0; \quad (\rho_0 + \rho) \frac{\partial \tilde{v}}{\partial t} + \tilde{v} \frac{\partial v}{\partial t} + C_0^2 \frac{\partial}{\partial x}(\rho_0 + \tilde{\rho}) = 0$$

$$\rho_0 \frac{\partial \tilde{v}}{\partial t} + C_0^2 \frac{\partial}{\partial x} \tilde{\rho} = 0; \quad \rho_0 \frac{\partial \tilde{v}}{\partial t} + C_0^2 \nabla \hat{\rho} = 0$$

Ск-ть звука. Ск-ть распр-я малых возм-ий.

(1) $\frac{\partial \tilde{\rho}}{\partial t} + \rho_0 \operatorname{div} \tilde{v} = 0$; $\rho_0 \frac{\partial \tilde{v}}{\partial t} + C_0^2 \operatorname{grad} \tilde{\rho} = 0$ (2) (уберём „~" для простоты)

$\frac{\partial}{\partial t}(1) \Rightarrow \frac{\partial^2 \rho}{\partial t^2} + \rho_0 \operatorname{div} \frac{\partial v}{\partial t} = 0$ (возьмём произв. по t от (1)). Возьмём div от (2)

$\Rightarrow \rho_0 \operatorname{div} \frac{\partial v}{\partial t} + C_0^2 \operatorname{div}(\operatorname{grad} \rho) = 0$; $\rho_0 \operatorname{div} \frac{\partial v}{\partial t} + C_0^2 \Delta \rho = 0 \Rightarrow \frac{\partial^2 \rho}{\partial t^2} = C_0^2 \Delta \rho$

$\operatorname{grad} \frac{\partial \rho}{\partial t} + \rho_0 \operatorname{grad}(\operatorname{div} v) = 0$; $(\operatorname{grad}(1))$; $\Leftrightarrow$ $\rho_0 \frac{\partial^2 v}{\partial t^2} + C_0^2 \operatorname{grad} \frac{\partial \rho}{\partial t} = 0$; $\left(\frac{\partial}{\partial t}(2)\right)$;

$\Rightarrow \frac{\partial^2 v}{\partial t^2} = c_0^2 \,\mathrm{grad}\,\mathrm{div}\, v$; $\mathrm{rot}\,\mathrm{rot}\, v = \mathrm{grad}\,\mathrm{div}\, v - \Delta v$; $\mathrm{rot}\, v = 0 \Rightarrow \mathrm{grad}\,\mathrm{div}\, v = \Delta v$;

$\frac{\partial^2 v}{\partial t^2} = c_0^2 \Delta v$; $\rho \to p \Rightarrow \frac{\partial^2 p}{\partial t^2} = c_0^2 \Delta p$; $\frac{\partial u}{\partial t^2} = c_0^2 \Delta u$, где $u = p, v, \rho$

← ур-е гиперб-го типа

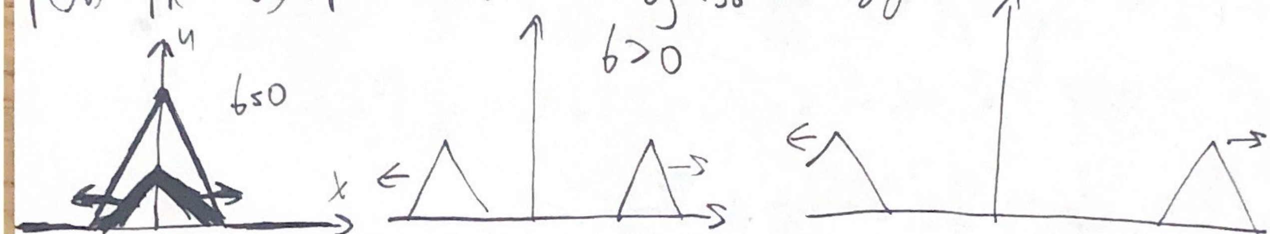
$u(x,0) = \varphi(x)$, $u_t(x,0) = \psi(x)$

$u(x,t)$ предст. в виде двух волн, распр. налево и направо со ск-тью c_0:

$u(x,t) = f_1(x - c_0 t) + f_2(x + c_0 t)$; $u(x,t) = \frac{\varphi(x - c_0 t) + \varphi(x + c_0 t)}{2} + \frac{1}{c_0} \int_{x - c_0 t}^{x + c_0 t} \psi(\xi)\, d\xi$

$\rho(x,t) = \rho_1(x - c_0 t) + \rho_2(x + c_0 t)$; $v(x,t) = v_1(x - c_0 t) + v_2(x + c_0 t)$ ^ ф-ла Деламбера

$p(x,t) = p_1(x - c_0 t) + p_2(x + c_0 t)$; $c_0^2 = \frac{\partial p}{\partial \rho}\Big|_{S_0}$ – адиабат. ск-ть звука



Выр-я для ск-ти звука для идеального газа

$\frac{p}{\rho^\gamma} = \frac{p_0}{\rho_0^\gamma} = A = \mathrm{const}$; $p = A\rho^\gamma$; $c_0 = \frac{\partial p}{\partial \rho}\Big|_{S = S_0}$; $p = \rho R T$; $\frac{\partial p}{\partial \rho} = A\gamma \rho^{\gamma-1} = \frac{p_0}{\rho_0^\gamma}\gamma \rho^{\gamma-1}\Big|_{\rho_0} =$

$= \frac{p_0}{\rho_0}\gamma = \gamma R T \Rightarrow \boxed{c_0^2 = \gamma R T}$; $\frac{\partial \rho}{\partial t} + \rho_0 \frac{\partial}{\partial x} v = 0$; $\rho = \rho_1(x - c_0 t) + \rho_2(x + c_0 t)$;

$\rho_0 \frac{\partial v}{\partial t} + c_0^2 \frac{\partial \rho}{\partial x} = 0$; $v = v_1(x - c_0 t) + v_2(x + c_0 t)$. Будем рассм. $\rho_1(x - c_0 t)$ и $v_1(x - c_0 t)$

ρ' v'

подставим в ур-я: $\xi = x - c_0 t \Rightarrow \frac{\partial \rho}{\partial t} = \frac{d\rho_1}{d\xi} \cdot \frac{d\xi}{dt} = \frac{d\rho_1}{d\xi}(-c_0)$;

$\frac{\partial \rho}{\partial x} = \frac{d\rho_1}{d\xi} \cdot \frac{\partial \xi}{\partial x} = \frac{d\rho_1}{d\xi} \Rightarrow \begin{cases} \rho_1'(-c_0) + \rho_0 v_1' = 0 \\ \rho_0(-c_0) v_1' + c_0^2 \rho_1' = 0 \end{cases} \Rightarrow \rho_0 v_1' - c_0 \rho_1' = 0$; $v_1(\xi), \rho_1(\xi)$

$\int_{\xi_0}^{\xi} (\rho_0 v_1' - c_0 \rho_1)\, d\xi = 0 \Rightarrow \rho_0 v_1(\xi) - c_0 \rho_1(\xi) \Rightarrow \frac{\rho_1}{\rho_0} = \frac{v_1}{c_0} \ll 1$;

$\rho_1 \ll \rho_0 \Rightarrow \frac{v_1}{c_0} \ll 1$, $v_1 \ll c_0$ (ск-ть звука)